2.3: A GENERAL POWER LOSS METHOD FOR ATTENUATION OF CAVITIES AND WAVEGUIDES

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The usual power loss method of evaluating the damping constant of cavities and the attenuation constant of waveguides, as caused by finite wall conductivity, breaks down in the case of degenerate modes and fails to predict the coupling between degenerate modes [1,2]. Papadopoulos has treated the problem by means of a perturbation method involving an expansion of the fields in terms of the modes for the ideal cavity or waveguide [2]. In this paper a variational formulation is presented that permits the eigenvalues for the lossy case to be readily computed. This formulation turns out to be a simple extension of the usual power loss method and in addition to giving the damping constant it also shows that there is an equal shift, in the resonant frequency in the case of a cavity, and in the phase constant for a waveguide. In addition the coupling between the degenerate modes is obtained. Furthermore it is shown that the new non-degenerate sets of coupled modes form an orthogonal set. The above properties all arise from the characteristics of the matrix eigenvalue problem which arises when the Rayleigh-Ritz technique is used in conjunction with the variational formulation of the boundary value problem.

For a cavity with N degenerate modes let $\bar{E}_n$, $\bar{H}_n$ be the field for the nth mode with resonant frequency ω_o when there are no losses. For finite losses let the field be represented by

$$\bar{E} = \sum_{1}^{N} a_n \bar{E}_n, \quad \bar{H} = \sum_{1}^{N} a_n \bar{H}_n$$

with resonant frequency ω. The variational formulation then leads to the following matrix eigenvalue problem:

$$|P - \Lambda W| = 0,$$

where P and W are N by N matrices with elements

$$P_{ij} = \frac{R_m}{2} \oint_S \bar{J}_i \cdot \bar{J}_j \, dS,$$

$$W_{ij} = \frac{\mu}{2} \int_V \bar{H}_i \cdot \bar{H}_j dV,$$

and

$$\Lambda = \frac{\omega_o^2 - \omega^2}{(1-j)\omega} .$$

$Z_m = (1+j)R_m$ is the surface impedance, and $\overline{J}_i = \overline{n} \times \overline{H}_i$ is the current on the cavity surfaces S for the ith mode. Since P and W are real and symmetric it follows that Λ must be real. Thus if we let $\omega = \omega_o + \Delta\omega$ where $|\Delta\omega| << \omega_o$ it follows that $\Lambda \approx -(2\Delta\omega)/(1-j)$ and hence $\Delta\omega = -\alpha + j\alpha$, i.e., a damping constant α is introduced and an equal decrease in the resonant frequency occurs. For each root Λ_i a solution for a set of coefficients a_n^i may be found. These determine the coupling between the original degenerate modes of the ideal cavity.

The case of the waveguide leads to a similar eigenvalue problem although the variational formulation is somewhat more involved than that for the cavity. If $\overline{E}_n, \overline{H}_n$ are the fields of the nth degenerate mode in a lossless guide then it is found that

$$|P - \Lambda W| = 0$$

where P and W are matrices with elements

$$P_{ij} = \frac{R_m}{2} \oint_C (\overline{n} \times \overline{H}_i) \cdot (\overline{n} \times \overline{H}_j^*) d\ell,$$

$$W_{ij} = \frac{1}{2} \int_S \overline{E}_i \times \overline{H}_j^* \cdot d\overline{S},$$

and $$\Lambda = \frac{\gamma^2 - \gamma_o^2}{\gamma_o(1+j)},$$

where γ_o is the propagation constant for the degenerate modes in the loss free guide and γ is the propagation constant for the lossy guide. Again since Λ must be real it follows that $\Delta\gamma = \gamma - \gamma_o = \alpha - j\alpha$ where α is the attenuation constant and also the change in β_o where $j\beta_o = \gamma_o$. For N degenerate modes N solutions for Λ exist as in the case of the cavity.

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1. S. Kuhn, "Calculation of Attenuation in Waveguides," Proc. IEE 93, Part IIIA, 663-678 (1946).

2. V. M. Papadopoulos, "Propagation of Electromagnetic Waves in Cylindrical Waveguides with Imperfectly Conducting Walls," Quart. J. Mech. and Appl. Math. 7, 325-334 (1954).